WHEN TO LEAVE A MONETARY UNION?

Frank Strobel

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When to Leave a Monetary Union?

Frank Strobel*

Using a two-country model of monetary union where policymakers minimize the continuous-time equivalent of a Barro-Gordon-type loss function, we examine the value of the option of monetary disintegration when the national preference parameters associated with an inflationary surprise follow correlated geometric Brownian motions. We derive the critical level of the ratio of these parameters that triggers a move to monetary disintegration and find that a country will be willing to return to monetary independence only if the other country’s relative inflation preferences are strictly, and potentially substantially, greater than a benchmark value depending on the cost of monetary disintegration alone.

QUAND QUITTER UNE UNION MONÉTAIRE?

À l’aide d’un modèle d’union monétaire à deux pays où les autorités politiques minimisent l’équivalent, en temps continu, d’une fonction de perte de type Barro-Gordon, on examine la valeur de l’option associée à l’éclatement d’une telle union. Les paramètres nationaux de préférence associés à une inflation surprise suivent des mouvements browniens géométriques corrélés. On dérive la valeur critique du ratio de ces paramètres au-delà de laquelle il y a désintégration monétaire : un pays aura tendance à vouloir retrouver son indépendance monétaire seulement si les préférences relatives en termes d’inflation de l’autre pays sont strictement (parfois considérablement) supérieures à une valeur de référence qui dépend uniquement du coût de désintégration monétaire.

Classification JEL: E5, F3

INTRODUCTION

When is it optimal for a country to leave a monetary union? For countries whose preferences over inflation differ, conventional wisdom suggests that any one of them will generally benefit from returning to monetary independence if the supranational preferences governing policymaking in the monetary union become less inflation averse than its own. This view is certainly too simplistic,
as the rapidly growing literature on irreversible investment under uncertainty shows us that the decision to invest in an irreversible project with uncertain payoffs can be profoundly affected when that investment can be delayed, as the option of waiting then typically has non-zero value and needs to be accounted for. Applying this particular methodology to a country’s decision of whether or not to return to monetary independence, thus interpreted as largely irreversible with uncertain benefits, should therefore allow a more rigorous understanding of the importance of relative inflation preferences in this context.

To investigate these issues in more detail, we use a simple two-country model of monetary union where policymakers minimize the continuous-time equivalent of a Barro-Gordon-type loss function over inflation, and examine the value of the option of monetary disintegration when the national preference parameters associated with an inflationary surprise follow correlated geometric Brownian motions. We derive the critical level of the ratio of these parameters that triggers a move to monetary disintegration and find that a country will generally be willing to return to monetary independence only if the other country’s inflation preferences are higher than its own by a factor strictly, and potentially substantially, greater than a benchmark value depending on the proportional cost of monetary disintegration alone. Higher uncertainty regarding these inflation preferences increases the value of the option of monetary disintegration and thereby raises the trigger value that prompts that option to be exercised. A higher discount rate (i.e. policymakers being more short-sighted) increases the opportunity cost of leaving the option of monetary disintegration unexercised for a further instant, and thus lowers the value of that option. The likelihood of the two countries’ inflation preference parameters drifting apart gets smaller the more correlated these are, having the same effect. Lastly, exercising the option of monetary disintegration becomes more onerous the higher the proportional cost associated with this move, raising the trigger value that prompts a return to monetary independence.

Section 2 now sets up the model and characterizes the optimal stopping problem involved, section 3 presents the solution and discusses our results, and section 4 concludes the paper.

**MODEL**

**General loss function**

We assume the policymaker’s objective involves the instantaneous loss rate

\[ l(b, \pi, t) = \frac{1}{2} \left[ \pi(t) \right]^2 - b(t) \left[ \pi(t) - \pi^c(t) \right] \]

1. See e.g. Dixit [1992], Pindyck [1991] or, more comprehensively, Dixit/Pindyck [1994].

2. A similar, but more general, framework is used in Strobel [2000] to study the decision of joining a monetary union.
where $\pi(t)$ and $\pi^e(t)$ represent inflation and expected inflation, respectively.\(^1\)

The time-varying inflation preference (or benefit) parameter $b(t) \geq 0$ follows a geometric Brownian motion with drift such that

$$db = ab\,dt + \sigma b\,dz$$

where $a$ and $\sigma^2$ are the respective instantaneous drift and variance rates, while $dz = \varepsilon(t)\sqrt{dt}$ is the increment of a Wiener process with $\varepsilon(t) \sim \text{NID}(0,1)$.

Restricting our analysis to a discretionary policy scenario, the policymaker’s choice problem is then to solve for the optimal feedback rule $\pi^*(b)$ that satisfies the loss function

$$L(b) = \min_{\pi} E_t \int_t^\infty l(b, \pi, \tau) e^{-\mu(\tau - t)} \, d\tau$$

where $\mu > 0$ is the discount rate, treating inflationary expectations $\pi^e(\tau)$ as given $\forall \tau \geq t$. Using the corresponding Bellman equation and applying Ito’s Lemma, we obtain

$$\pi^*(b) = b(t)$$

as the optimal feedback rule in question.

Imposing rational expectations such that $\pi^e(\tau) = \pi(\tau), \forall \tau \geq t$ at this stage and using standard properties\(^2\) of geometric Brownian motion to simplify, the resulting loss function becomes

$$L(b) = \frac{1}{2(\mu - a - \sigma^2)} [b(t)]^2 \quad (1)$$

in equilibrium, as long as $2a + \sigma^2 - \mu < 0$ for convergence.

**Monetary independence & integration**

We characterize the case of monetary independence such that the national policymaker’s inflation preference parameter $b_i(t)$ in each country $i = 1, 2$ follows a geometric Brownian motion without drift

$$db_i = \sigma b_i \, dz_i$$

where $\sigma \geq 0$ and $E_t(dz_1 \, dz_2) = \rho dt$, with $\rho$ the coefficient of correlation between the processes $z_i$ and thus $-1 \leq \rho \leq 1$. Using equation (1) above, in equilibrium the respective loss functions $L(b_i)$ then become

$$L(b_i) = \frac{1}{2(\mu - \sigma^2)} [b_i(t)]^2 \quad (2)$$

as long as $\sigma^2 - \mu < 0$, where we assume a common discount rate $\mu$.

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\(^1\) This adapts the discrete-time setup in Barro/Gordon [1983] to a continuous-time environment; a similar, but more general, framework is used in Strobel [2000].

\(^2\) See e.g. Dixit ([1993], eq. (2.2)).
For the monetary integration case we assume that the supranational policymaker’s inflation preference parameter is determined symmetrically as 

\[
b_{12}(t) = \sqrt{b_1(t) b_2(t)}
\]

with the constituent national inflation preference parameters \(b_i(t)\) evolving as above. Using Ito’s Lemma and drawing again on equation (1), in equilibrium the loss function \(L(b_{12})\) then becomes

\[
L(b_{12}) = \frac{1}{2(\mu - \rho \sigma^2)} b_1(t) b_2(t)
\]

as long as \(\rho \sigma^2 - \mu < 0\).

**Optimal stopping problem**

Starting from a situation of monetary integration between countries 1 and 2, the decision of, say, country 1 on whether or not to return to monetary independence involves solving the Bellman equation for the optimal stopping problem

\[
F(L_{12}, L_1) = \max \left\{ L_{12} - (1 + \tau) L_1, \frac{1}{\mu dt} E_t \left[ dF(L_{12}, L_1) \right] \right\}
\]

where \(F(L_{12}, L_1)\) is the value to country 1 of the option of monetary disintegration, \(\tau \geq 0\) is the proportional cost of returning to monetary independence, and \(L_{12} - (1 + \tau) L_1\) is the (net) expected discounted benefit of such a move\(^1\).

In the continuation region, where postponing monetary disintegration for a further instant \(dt\) is optimal, the relevant Bellman equation is then just

\[
\mu F(L_{12}, L_1) = \frac{1}{dt} E_t \left[ dF(L_{12}, L_1) \right]
\]

Applying Ito’s Lemma and noting that the value function \(F(L_{12}, L_1)\) should be homogeneous of degree 1\(^2\), so that \(F(L_{12}, L_1) = L_1 f(\Gamma)\) where \(\Gamma \equiv \frac{L_{12} b_2}{L_1 b_1}\), we obtain

\[
\sigma^2(1 - \rho) \Gamma^2 \frac{\partial^2 f}{\partial \Gamma^2} - \sigma^2(1 - \rho) \frac{\partial f}{\partial \Gamma} + (\sigma^2 - \mu) f = 0
\]

as the differential equation characterizing the value function \(F(L_{12}, L_1)\) in that region. The corresponding value-matching condition

\[
f(\Gamma^*) = \Gamma^* - (1 + \tau)
\]

and smooth-pasting condition

\[
\frac{\partial f(\Gamma^*)}{\partial \Gamma} = 1
\]

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1. We use \(L_i \equiv L(b_i)\) etc. for ease of notation.
2. This adopts the solution strategy in Dixit/Pindyck ([1994], p. 210).
3. Thus, \(\Gamma = \frac{\mu - \sigma^2}{\mu - \rho \sigma^2} b_2(t)\) from equations (2) and (3).
then allow derivation of the value function $F(L_{12}, L_1)$ and the boundary $(L_{12}^*, L_1^*)$ in $(L_{12}, L_1)$ space that separates the region where the option of monetary disintegration remains unexercised from the one where exercise of that option is immediate.

**SOLUTION & DISCUSSION**

We observe that the differential equation (4) becomes degenerate for the non-stochastic case where $\sigma = 0$, and for the symmetric scenario where $\rho = 1$. In both cases we obtain $f(\Gamma) = 0$, so that country 1’s option of monetary disintegration has zero value throughout. Note also that $\Gamma^* = 1 + \tau$ then follows from value-matching condition (5), giving $\frac{b_2^*}{b_1^*} = 1 + \tau$ from the definition of $\Gamma$. Thus, when there is no uncertainty about the evolution of country 1 and 2’s inflation preferences, or their inflation preferences are perfectly correlated, the trigger value of relative inflation preference parameters $b_2^* / b_1^*$ depends solely on the proportional cost of monetary disintegration $\tau$, with country 1 generally willing to return to monetary independence only when country 2’s inflation preferences are higher than its own by at least a factor of $1 + \tau$.

Turning now to the more interesting non-degenerate case where $\sigma > 0$ and $\rho < 1$, we solve equation (4) by standard methods, using value-matching condition (5), smooth-pasting condition (6) as well as the additional boundary condition $f(0) = 0$, and obtain

$$f(\Gamma) = (\beta_1 - 1) \beta_i - 1 (1 + \tau)^{1 - \beta_1} \beta_1 \beta_i \Gamma^\beta_i$$

where the (real) root $\beta_i$ is given by

$$\beta_i = 1 + \frac{1}{\sigma} \sqrt{\frac{\mu - \rho \sigma^2}{1 - \rho}} > 1$$

We further obtain

$$\Gamma^* = (1 + \tau) \left( 1 + \sigma \sqrt{\frac{1 - \rho}{\mu - \rho \sigma^2}} \right)$$

1. That is the value of $\frac{b_2^*}{b_1^*}$ separating the region in $(b_1, b_2)$ space where the option of monetary disintegration remains unexercised (i.e. for $\frac{b_2}{b_1} < \frac{b_2^*}{b_1^*}$) from the one where exercise of that option is immediate (i.e. for $\frac{b_2}{b_1} \geq \frac{b_2^*}{b_1^*}$).

2. Note that zero is an absorbing barrier for the geometric Brownian motion $\Gamma$. 

as the critical value \( \Gamma^* \). From the definition of \( \Gamma \) it then follows that

\[
\frac{b_2^*}{b_1^*} = \frac{1 + \tau}{1 - \sigma \sqrt{\frac{1 - \rho}{\mu - \rho \sigma^2}}}
\]

(7)

is the trigger value of relative inflation preference parameters \( \frac{b_2^*}{b_1^*} \) in the non-degenerate case, depending both on the value of the option of monetary disintegration and the proportional cost of returning to monetary independence. In particular, it can be shown that \( \frac{b_2^*}{b_1^*} > 1 + \tau \), so that country 1 will generally be willing to return to monetary independence only if country 2’s inflation preferences are higher than its own by a factor strictly greater than \( 1 + \tau \), the benchmark value for the degenerate cases of \( \sigma = 0 \) and \( \rho = 1 \) discussed above.

Intuitively, country 1’s option of monetary disintegration has positive value in the non-degenerate case and will therefore be exercised only at a point where, in the jargon of financial options, it is sufficiently “in-the-money.”

Examining more closely the directional impact of changes in \( \sigma, \mu, \rho \) and \( \tau \) on \( \frac{b_2^*}{b_1^*} \), we obtain

\[
\frac{\partial}{\partial \sigma} \frac{b_2^*}{b_1^*} = \frac{\Theta (1 + \tau) (1 - \rho)}{(\mu - \sigma^2)^2} > 0
\]

\[
\frac{\partial}{\partial \tau} \frac{b_2^*}{b_1^*} = \frac{1}{1 - \sigma \sqrt{\frac{1 - \rho}{\mu - \rho \sigma^2}}} > 0
\]

\[
\frac{\partial}{\partial \mu} \frac{b_2^*}{b_1^*} = -\frac{\Theta (1 + \tau) (1 - \rho) \sigma}{2 (\mu - \sigma^2)^2} < 0
\]

\[
\frac{\partial}{\partial \rho} \frac{b_2^*}{b_1^*} = -\frac{\Theta (1 + \tau) \sigma}{2 (\mu - \sigma^2)} < 0
\]

where \( \Theta \equiv 2 \sigma + \frac{\sigma^2 (1 - \rho) + \mu - \rho \sigma^2}{\sqrt{(1 - \rho) (\mu - \rho \sigma^2)}} > 0. \)

We note that \( \frac{b_2^*}{b_1^*} \) is increasing in \( \sigma \), a result familiar from the standard option pricing literature, as higher uncertainty regarding country 1 and 2’s inflation preferences increases the value of the option of monetary disintegration and thereby raises the trigger value that prompts that option to be exercised. The
Figure 1. Trigger value (\( \tau = 0.05, \mu = 0.1 \))

Figure 2. Trigger value (\( \tau = 0.05, \rho = 0 \))

Figure 3. Trigger value (\( \rho = -0.5, \mu = 0.1 \))
trigger value \( \frac{b_2^*}{b_1^*} \) is also increasing in \( \tau \), as exercising the option of monetary disintegration becomes more onerous the higher the proportional cost associated with this move. Increasing \( \mu \) leads to lower levels of \( \frac{b_2^*}{b_1^*} \), as a higher discount rate (i.e. policymakers being more short-sighted) raises the opportunity cost of leaving the option of monetary disintegration unexercised for a further instant, and thus decreases the value of that option. The trigger value \( \frac{b_2^*}{b_1^*} \) is similarly decreasing in \( \rho \), as the likelihood of the two countries’ inflation preference parameters drifting apart gets smaller the more correlated these are, thereby decreasing the value of the option to return to monetary independence.

These qualitative results are illustrated in figures 1–4, where we graph the trigger value \( \frac{b_2^*}{b_1^*} \) for different parameter combinations of \( \sigma, \mu, \rho \) and \( \tau \). While serious parameterization of the model may be somewhat ambitious due to its relative simplicity, it is nevertheless worthwhile noting the substantial magnitudes of \( \frac{b_2^*}{b_1^*} \) that can arise, and the relative dominance of the value of the option of monetary disintegration versus the proportional cost of returning to monetary independence in the determination of the trigger value \( \frac{b_2^*}{b_1^*} \).

CONCLUSION

Using a simple two-country model of monetary union where policymakers minimize the continuous-time equivalent of a Barro-Gordon-type loss function over inflation, we examined the value of the option of monetary disintegration when the national preference parameters associated with an inflationary surprise follow correlated geometric Brownian motions. We derived the critical level of the ratio of these parameters that triggers a move to monetary disintegration and

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found that a country will generally be willing to return to monetary independence only if the other country’s inflation preferences are higher than its own by a factor strictly, and potentially substantially, greater than a benchmark value depending on the proportional cost of monetary disintegration alone. Higher uncertainty regarding these inflation preferences increases the value of the option of monetary disintegration and thereby raises the trigger value that prompts that option to be exercised. A higher discount rate (i.e. policymakers being more short-sighted) increases the opportunity cost of leaving the option of monetary disintegration unexercised for a further instant, and thus lowers the value of that option. The likelihood of the two countries’ inflation preference parameters drifting apart gets smaller the more correlated these are, having the same effect. Lastly, exercising the option of monetary disintegration becomes more onerous the higher the proportional cost associated with this move, raising the trigger value that prompts a return to monetary independence.

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